Recommended Study Method

Do lots and lots of odd number problems in the book within sections 2.1 – 2.4. If you run into problem, ask (me or a friend)! Redo various worksheet problems from function composition worksheet.

Reading this document should only be a portion of your study time.

# Chapter 2 Review Guide

## Function Transformation

- Function Composition
  - $f \circ g(x) = f(g(x))$
    - Ex. $g(x) = x + 2$ and $f(x) = x^2 + 2x$
      $f(g(x)) = (x + 2)^2 + 2(x + 2) =$ (simplify…)
- Transformations

| | Vertical | Horizontal |
|---|---|---|
| Translation<br>k>0: up<br>h>0: right | $f(x) + k$<br>(x,y) → (x,y+k) | $f(x - h)$<br>(x,y) → (x+h,y) |
| Reflection<br>x-axis: vertical<br>y-axis: horizontal | $-f(x)$<br>(x,y) → (x,-y) | $f(-x)$<br>(x,y) → (-x,y) |
| Stretch / Shrink<br>(a>1) (a<1) | $af(x)$<br>(x,y) → (x,ay) | $f(\frac{1}{a}x)$<br>(x,y) → (ax,y) |

Given $f(x) = x^2 + 5x$

a) Translate f(x) up by 5

b) Then, reflect it over the y-axis

d) Then, horizontally shrink it by 1/2

e) Then, translate to left by 4

Do you get?

$4x^2 + 22x + 29$

- Translation/Reflections: Rigid transformation (pre/post graphs are "congruent")
- Order of Transformation
  - Be able to determine if two transformations can be done in reverse order
  - Basic rule
    - All stretches and reflections are commutable ("reorderable")
    - All translations are commutable
    - A horizontal transformation + vertical transformation can commutable
    - ONLY NON-COMMUTABLE TRANSFORMATION
      - Vertical transformation (stretch/reflection) + Vertical translation
      - Horizontal transformation (stretch/reflection) + Horizontal translation

## Quadratic Function – Equation Forms and Properties

- **Know this chart!!!**

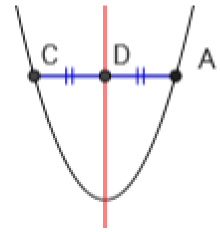
| | **Vertex** | **Standard** | **Intercept** |
|---|---|---|---|
| **Equation** | $f(x) = a(x - h)^2 + k$ | $f(x) = ax^2 + bx + c$ | $f(x) = a(x - p)(x - q)$ |
| **Vertex** | $(h, k)$ | $(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right))$ | $(\frac{p+q}{2}, f\left(\frac{p+q}{2}\right))$ |
| **Axis (Line) of symmetry** | $x = h$ | $x = -\frac{b}{2a}$ | $x = \frac{p+q}{2}$ |
| **y-intercept** | | $(0, c)$ | |
| **x-intercepts** | | | $(p, 0)$ & $(q, 0)$ |

Given any equation, you should be able to:

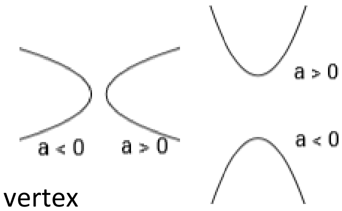
- Find vertex
- Find min/max
- Find y-intercepts (plug in x=0)
- Find increasing and decreasing intervals
- Find focus and directrix
- Find axis of symmetry

## Parabola Properties

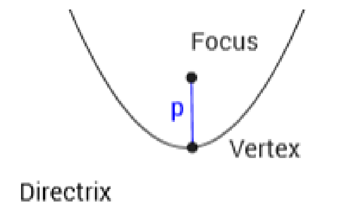
- Definition of Parabola
  - Set of points equidistant from a point (focus) and a line (directrix)
- Properties of Parabolas
  - Symmetric across the Axis of Symmetry
    - Any point of the parabola has a reflected point across the A.O.S – nice and useful‼




  - Direction
    - Up (or Right): a > 0
    - Down (or Left): a < 0




  - Min/Max
    - Min/Max is <u>always</u> at the vertex
    - Up parabolas – have minimum
    - Down parabolas – have maximum
  - Focus – point of focus on the inside of the parabola
    - p is a "signed" <u>distance</u> of the vertex to the focus
      - p > 0 for up and right parabolas
      - p < 0 for down and left parabolas
    - $a = \frac{1}{4p}$ and equivalently $p = \frac{1}{4a}$ ← Given a, you can find p. Given p, you can find a‼




  - Directrix – the line that is "outside" parabola a distance of p from the vertex
  - Increasing/Decreasing Intervals
    - The question is: "What are the values of x in which f(x) are increasing?"
    - Ex. $y = 2(x-4)^2 + 5$ ← vertex is at (4,5).
      - Decreasing: (-∞ , 4) Increasing: (4, ∞) since this is an "upward" parabola

## Left/Right Parabolas

- Fundamentally, everything is "opposite"
  - Swap the x and the y's
  - Swap the h and the k's
  - What was <u>vertical</u> transformations are now <u>horizontal</u> transformations
- Vertex form
  - $x = a(y-k)^2 + h$ ← NOTE: the h and k's are swapped‼! Be careful on this‼
  - Remember $a = \frac{1}{4p}$ …

## Modelling Quadratic Functions

- Determining if data is quadratic
  - Plot points and look at it ‼
  - Common Differences
    - If 1st difference is common, then linear
    - If 2nd difference is common, the quadratic

26,900 29,025 30,600 31,625 32,100 32,025 31,400

2125 1575 1025 475 −75 −625

−550 −550 −550 −550 −550

Always subtract in the same "direction" L→R or R→L

- Writing quadratic models from data points
  - Concept: Through any 3 non-colinear points on a plane, there is exactly 1 parabola that goes through the 3 points
  - Methods for each form
    - Intercept - $y = a(x - p)(x - q)$ → plug in p & q (from intercepts), x & y (from point) to solve for *a*
    - Vertex - $y = a(x - h)^2 + k$ → plug in h & k (from vertex), x & y (from point) to solve for *a*
    - Standard - $y = ax^2 + bx + c$ → plug x & y for each 3 points, creating 3x3 system of equations to solve for *a*, *b* and *c*

## Average Rate of Change

- For any f(x), average rate of change from x = a to b is $\frac{f(b)-f(a)}{b-a}$ ← slope of the line between the two points!!